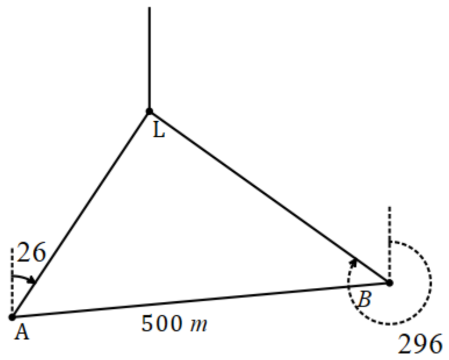
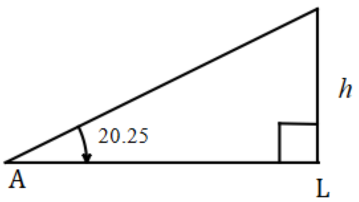
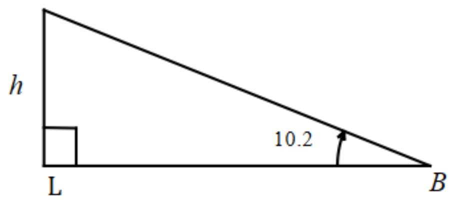
7 From a point A, a lighthouse is on a bearing of 026°T and the top of the lighthouse is at angle of elevation of 20.25°. From a point B, the lighthouse is on a bearing of 296°T and the top of the lighthouse is at angle of elevation of 10.20°. If A and B are 500 metres apart, find the height of the lighthouse, correct to the nearest metre.



L is the base of the lighthouse





Angle of elevation from A to the top of the lighthouse is 20.25°.

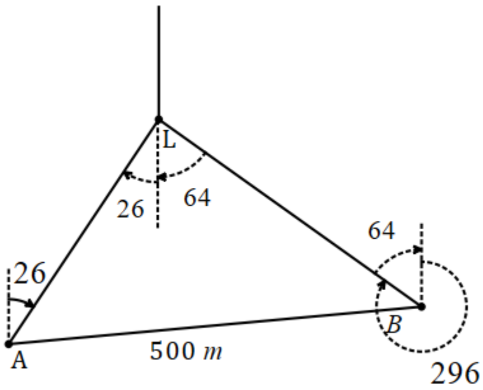
Angle of elevation from B to the top of the lighthouse is 10.2°.

Using the above two triangles,

$$\tan 20.25 = \frac{h}{AL}$$

$$\tan 10.2 = \frac{h}{BL}$$

Hence $AL = \frac{h}{\tan 20.25}$ and $BL = \frac{h}{\tan 10.2}$



The dashed north lines are parallel.

Hence by alternate angles in parallel lines we can find $\angle ALD = 26° + 64° = 90°$

Now we can use Pythagoras.

$$500^2 = (AL)^2 + (BL)^2$$

$$500^2 = \left(\frac{h}{\tan 20.25}\right)^2 + \left(\frac{h}{\tan 10.2}\right)^2$$

$$500^2 = \frac{h^2}{\tan^2 20.25} + \frac{h^2}{\tan^2 10.2}$$

$$500^2(\tan^2 20.25)(\tan^2 10.2) = h^2(\tan^2 10.2) + h^2\,(\tan^2 20.25)$$

$$500^2(\tan^2 20.25)(\tan^2 10.2) = h^2(\tan^2 10.2 + \tan^2 20.25)$$

$$h^2 = \frac{500^2(\tan^2 20.25)(\tan^2 10.2)}{\tan^2 10.2 + \tan^2 20.25}$$

$$h = 80.9m$$