

OCR 4726 June 2009 Question 6

$$\text{If } \int_0^1 \frac{1}{\sqrt{16+9x^2}} dx + \int_0^2 \frac{1}{\sqrt{9+4x^2}} dx = \ln a, \text{ find } a \text{ exactly.}$$

$$\text{Let } x = \frac{4}{3} \tan \theta$$

$$\begin{aligned}\frac{dx}{d\theta} &= \frac{4}{3} \sec^2 \theta \\ dx &= \frac{4}{3} \sec^2 \theta d\theta\end{aligned}$$

$$\text{When } x = 0$$

$$\begin{aligned}0 &= \frac{4}{3} \tan \theta \\ \theta &= 0\end{aligned}$$

$$\text{When } x = 1$$

$$\begin{aligned}1 &= \frac{4}{3} \tan \theta \\ \theta &= \tan^{-1}\left(\frac{3}{4}\right)\end{aligned}$$

$$\text{Let } x = \frac{3}{2} \tan \theta$$

$$\begin{aligned}\frac{dx}{d\theta} &= \frac{3}{2} \sec^2 \theta \\ dx &= \frac{3}{2} \sec^2 \theta d\theta\end{aligned}$$

$$\text{When } x = 0$$

$$\begin{aligned}0 &= \frac{3}{2} \tan \theta \\ \theta &= 0\end{aligned}$$

$$\text{When } x = 2$$

$$\begin{aligned}2 &= \frac{3}{2} \tan \theta \\ \theta &= \tan^{-1}\left(\frac{4}{3}\right)\end{aligned}$$

$$\tan^{-1}\left(\frac{3}{4}\right)$$

$$\int_0^{\tan^{-1}\left(\frac{3}{4}\right)} \frac{1}{\sqrt{16 + 9\left(\frac{16}{9}\tan^2 \theta\right)}} \times \frac{4}{3} \sec^2 \theta d\theta + \int_0^{\tan^{-1}\left(\frac{4}{3}\right)} \frac{1}{\sqrt{9 + 4\left(\frac{9}{4}\tan^2 \theta\right)}} \times \frac{3}{2} \sec^2 \theta d\theta$$

$$\tan^{-1}\left(\frac{3}{4}\right)$$

$$\int_0^{\tan^{-1}\left(\frac{3}{4}\right)} \frac{1}{\sqrt{16 + 16\tan^2 \theta}} \times \frac{4}{3} \sec^2 \theta d\theta + \int_0^{\tan^{-1}\left(\frac{4}{3}\right)} \frac{1}{\sqrt{9 + 9\tan^2 \theta}} \times \frac{3}{2} \sec^2 \theta d\theta$$

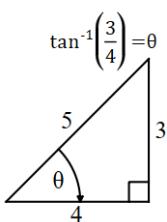
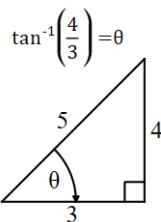
$$\tan^{-1}\left(\frac{3}{4}\right)$$

$$\int_0^{\tan^{-1}\left(\frac{3}{4}\right)} \frac{1}{4 \sec \theta} \times \frac{4}{3} \sec^2 \theta d\theta + \int_0^{\tan^{-1}\left(\frac{4}{3}\right)} \frac{1}{3 \sec \theta} \times \frac{3}{2} \sec^2 \theta d\theta$$

$$\tan^{-1}\left(\frac{3}{4}\right)$$

$$\int_0^{\tan^{-1}\left(\frac{3}{4}\right)} \frac{\sec \theta}{3} d\theta + \int_0^{\tan^{-1}\left(\frac{4}{3}\right)} \frac{\sec \theta}{2} d\theta$$

$$\left[ \frac{1}{3} (\ln |\sec \theta + \tan \theta|) \right]_{0}^{\tan^{-1}\left(\frac{3}{4}\right)} + \left[ \frac{1}{2} (\ln |\sec \theta + \tan \theta|) \right]_{0}^{\tan^{-1}\left(\frac{4}{3}\right)}$$



$$\frac{1}{3} (\ln \left| \frac{5}{4} + \frac{3}{4} \right| - \ln |1 - 0|) + \frac{1}{2} (\ln \left| \frac{5}{3} + \frac{4}{3} \right| - \ln |1 - 0|)$$

$$\frac{1}{3} \ln 2 + \frac{1}{2} \ln 3$$

$$\ln a = \ln \left( 2^{\frac{1}{3}} \times 3^{\frac{1}{2}} \right)$$

$$a = 2^{\frac{1}{3}} \times 3^{\frac{1}{2}}$$