Techniques of Integration – Trigonometric Substitution.

Remember the trigonometric identities

$$1 - \sin^2 x = \cos^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$\sec^2 x - 1 = \tan^2 x$$

We can use the identities to integrate functions like:

$$\sqrt{a^2 - x^2}$$
$$\sqrt{a^2 + x^2}$$
$$\sqrt{x^2 - a^2}$$

For example

1. $\int \frac{dx}{\sqrt{4-x^2}}\, dx$

   Let $x = 2\sin u$

   $u = \sin^{-1}\frac{x}{2}$

   $\frac{dx}{du} = 2\cos u$

   $dx = 2\cos u\, du$

> The aim of the substitution is to get one of the trig identities.
>
> Choose $2\sin u$ because you will then get $4 - 4\sin^2 u = 4(1 - \sin^2 u) = 4\cos^2 u$

$$\int \frac{dx}{\sqrt{4 - x^2}} dx = \int \frac{2\cos u\, du}{\sqrt{4 - 4\sin^2 u}}$$

$$= \int \frac{2\cos u\, du}{\sqrt{4(1 - \sin^2 u)}}$$

$$= \int \frac{2\cos u\ \ du}{2\cos u}$$

$$= \int 1\, du$$

$$= u + c$$

$$= \sin^{-1}\frac{x}{2} + c$$

2. $\int \frac{dx}{25+x^2}$

3. $\int \frac{1}{\sqrt{x^2-4}}\, dx$